Chapter 2

CHARACTERISTICS OF SYNCHROTRON RADIATION

2.1 Introduction

The radiation is characterized in general by the following terms: spectral range, photon flux, photon flux density, brilliance, and the polarization. The photon flux is the overall flux collected by an experiment and reaching the sample, the photon flux density is the flux per area at the sample and the brilliance is the flux per area and opening angle. In the following chapter the formulas for the calculation of these terms of the synchrotron radiation emitted from a stored beam in the bending magnet, wiggler and undulator are compiled.

Many authors have established the theory of synchrotron radiation. Today most of the calculations are using the results of the Schwinger theory [1]. A relativistic electron running around an orbit emits a radiation characteristic as given in figure (2.1):

\[
\frac{d^2\Phi}{d\Theta d\Psi} = \frac{3\alpha}{4\pi^2} \frac{\Delta\omega I}{\omega e^2} \gamma^2 (1 + X^2) \left[ K_{2/3}^2 (\xi) + \frac{X^2}{1 + X^2} K_{1/3}^2 (\xi) \right]
\]

(2.1)

where:

- \(\Phi\) = photon flux (number of photons per second).
- \(\Theta\) = observation angle in the horizontal plane.
- \(\Psi\) = observation angle in the vertical plane.
- \(\alpha\) = fine structure constant = (1/137).

Figure 2.1: Characteristics of the synchrotron radiation emitted by a relativistic electron moving on a circle.
**CHARACTERISTICS OF SYNCHROTRON RADIATION**

\[ \gamma = \text{electron energy / mec}^2 (\text{me = electron mass, } c = \text{velocity of light}). \]

\[ \omega = \text{angular frequency of photons} \ (\hbar \omega = \text{photon energy} = \epsilon). \]

\[ I = \text{beam current}. \]

\[ e = \text{electron charge} = 1.601 \times 10^{-19} \text{coulomb}. \]

\[ \gamma = \omega / (\omega c = \epsilon / \epsilon e (\omega e = \text{critical frequency} = 3\gamma^3 / 2\rho)). \]

\[ \epsilon e = \text{critical photon energy} (= 3\hbar \gamma \beta / 2\rho). \]

\[ \rho = \text{radius of instantaneous curvature of electron trajectory} = E / e c B \text{ in practical units}, \]

\[ \rho(m) = 3.3356 \times (E / \text{GeV}) / (B/T). \]

\[ c = \text{speed of light} = 2.9979 \times 10^8 \text{m/s}. \]

\[ E = \text{electron beam energy}. \]

\[ B = \text{magnetic field strength}. \]

\[ \epsilon e = \hbar \omega e [\epsilon e(\text{keV}) = 0.665 \times (E / \text{GeV})^2 * (B/T)]. \]

\[ X = \gamma \psi (\text{normalized angle in the vertical plane}). \]

\[ \xi = \gamma (1 + X^2) / 2. \]

The subscripted K’s in equation (2.1) are modified Bessel functions of the second kind. Equation (2.1) is the basic formula for the calculation of the characteristics of the synchrotron radiation. The polarization is given by the two terms within the square brackets.

### 2.2 Radiation from a Bending Magnet

The photon flux of the synchrotron radiation from the bending magnet is given by equation (2.1), if integrated over the whole vertical angle. In the horizontal plane the emitted cone is constant and therefore the photon flux is proportional to the accepted angle \( \theta \) in the horizontal plane:

\[
\frac{d\Phi(y)}{d\theta} = 2.458 \cdot 10^{13} \text{ Photons s}^{-1} \text{BW mrad}^{-1} \delta y \left( E / \text{GeV} \right) \cdot (I / A) \cdot (\theta / \text{mrad}) \cdot G_1 \left( \frac{\epsilon}{\epsilon_e} \right)
\]

According to Equation (2.2), the photon flux is proportional to the beam current, the energy and the normalized function \( G_1(\epsilon / \epsilon e) \) which depends only from the critical photon energy. The equation for the critical photon energy is given in (2.3).

\[ \epsilon_e = 0.655 \text{keV} \cdot (E / \text{GeV})^2 \cdot (B/T) \]

The fluxes emitted within the bending magnet from a stored electron beam at different energies are presented in figure (2.2). The photon flux increases according to equation (2.2) with the energy and it is shifted to higher photon energies because of the larger critical energy. The photon flux density from the bending magnets for the same electron beam is presented in figure (2.3). Because the cross section of the beam decreases with the energy, the flux density increases with smaller energies.

The intensity of the synchrotron radiation in the middle of the radiation cone (\( \theta = 0 \) and \( \psi = 0 \)) is given by the following formula (central intensity):

\[
\frac{d^2\Phi(y)}{d\theta d\psi} = 1.326 \cdot 10^{13} \text{ Photons s}^{-1} \text{mrad}^{-2} \delta y \left( E / \text{GeV} \right)^2 \cdot (I / A) \cdot H_2 \left( \frac{\epsilon}{\epsilon_e} \right)
\]

Because the radiation cone is getting narrower with higher energy, the central intensity is proportional the square of the energy. The spectral dependency is given by the normalized synchrotron function \( H_2(y) = H_2(\epsilon / \epsilon e) \). From the definition of the flux (equation (2.2)) and the central intensity (equation (2.4)) the vertical opening angle of the synchrotron radiation is given by:

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2-2
Figure 2.2: The photon flux emitted within the bending magnets of SESAME from electron beams with different energies.

Figure 2.3: The photon flux density emitted within the bending magnet of SESAME from electron beams with different energies.

\[
\sigma_\psi = \frac{1}{\sqrt{2\pi}} \cdot \frac{\langle d\Phi \rangle}{d\theta} \bigg|_{\psi=0} = \sqrt{\frac{2\pi}{3}} \cdot \frac{1}{\gamma} \cdot \frac{G_1(y)}{H_2(y)} = 0.7395\, mrad \cdot \frac{1}{E/GeV} \cdot \frac{G_1(y)}{H_2(y)} \]

(2.5)

The opening angle of the synchrotron radiation from the bending magnet of SESAME for a stored beam with different energies is presented in figure (2.4):
The opening angle at the critical photon energy \( y=1 \) or \( \varepsilon = \varepsilon_c \) is, according to equation (2.5):

\[
\sigma_y \left( y = 1 \right) = \frac{0.331 \text{mrad}}{(E/\text{GeV})} \tag{2.6}
\]

For a 2.5 GeV machine the corresponding angle is 0.132 mrad.

The brilliance of the synchrotron radiation from a bending magnet is given by the central intensity divided by the cross section of the beam:

\[
Br = \frac{\langle \frac{d^2\Phi}{d\theta d\psi} \rangle(\psi = 0)}{2\pi \sum_x \sum_y} \tag{2.7}
\]

where

\[
\sum_x = \left[ \varepsilon_x \beta_x + \eta_x^2 \sigma_x^2 + \sigma_z^2 \right]^{1/2} \quad \text{and} \quad \sum_y = \left[ \varepsilon_y \beta_y + \sigma_y^2 + \varepsilon_y \gamma_y \sigma_y^2 \sigma_y^2 \right]^{1/2} \tag{2.8}
\]

with

- \( \varepsilon_x \) (\( \varepsilon_y \)) is the electron beam emittance in the horizontal (vertical) plane,
- \( \beta_x \) (\( \beta_y \)) is the electron beam beta function in the horizontal (vertical) plane,
- \( \eta_x \) is the dispersion function in the horizontal plane,
- \( \sigma_E \) is the rms value of the relative energy spread,
- \( \gamma_y \) is a Twiss parameter in the vertical plane,
- \( \sigma_y \) is the rms value of the radiation opening angle,
- \( \sigma_r = \lambda/(4\pi \sigma_y) \) is the diffraction limited source size,
- \( \lambda \) is the observed photon wavelength.

At a photon energy of 10 keV the corresponding photon wavelength is 0.124 nm and the opening angle is smaller than 0.1 mrad (see figure (2.4)). Both figures result in a diffraction limited source size of \( \sigma_r = 1.24 \mu\text{m} \). The term \( \varepsilon_\psi / \sigma_y \) gives a cross section of 2 \( \mu\text{m} \) and \( \varepsilon_\psi \gamma_2 / \sigma_y \) has a value between \( 1 \times 10^{-3} \) and \( 4 \times 10^{-3} \). These factors are at least one order of magnitude smaller than the beam cross section \( \varepsilon_x \) and \( \sigma_z \), hence the overall cross sections in equation (2.8) reduces for the storage ring SESAME to:
\[
\sum_s = \left[ e_s \beta_s + \eta_s^2 \sigma_s^2 \right]^{1/2}, \quad \sum_y = \left[ e_y \beta_y \right]^{1/2}
\] (2.9)

And the brilliance of the synchrotron radiation from the bending magnet of a non-diffraction limited light source is given by:

\[
Br_{\text{Magnet}} = \frac{\left\langle \frac{d^2 \phi}{d \psi} \right\rangle (\psi = 0)}{2\pi \left[ e_x \beta_x + \eta_x^2 \sigma_x^2 \right]^{1/2} \cdot \left[ e_y \beta_y \right]^{1/2}} = \frac{\left\langle \frac{d^2 \Phi}{d \psi} \right\rangle (\psi = 0)}{2\pi \sigma_x \sigma_y}
\] (2.10)

The brilliance of the synchrotron radiation emitted within the bending magnets at SESAME for different energies are presented in figure (2.5):

The critical photon energies of the different energies are: 2.5 GeV \(\equiv \epsilon_c=5.73\) keV, 2.0 GeV \(\equiv \epsilon_c=2.93\) keV, 1.5 GeV \(\equiv \epsilon_c=1.24\) keV and 1.0 GeV \(\equiv \epsilon_c=0.37\) keV. From the above figures it follows that the brilliance has its maximum around the critical photon energy and it has roughly the same value.

### 2.3 Radiation from a Wiggler

The wiggler is a special magnet with alternating directions of the magnetic field and the trajectory of an electron beam through a wiggler is like a snake, it is a sinusoidal oscillation. The arrangement of the magnets in a so-called “Hybrid Design (HYB)” is given in figure (2.6). The arrows in figure (2.6) symbolize the direction of the magnetic field in the different materials. The green and the yellow blocks are special permanent magnets and the brown blocks are made of magnetic steel. The trajectory of the electron beam in such a wiggler is presented in figure...
(2.7) (blue line). The red arrows symbolize the emitted radiation and it follows that an overlapping of the radiation cone will occur.

\[ X_0 = \frac{1}{2\pi} \frac{K}{\gamma} \cdot \lambda_p^2 = \frac{8.13 \cdot 10^{-5} \cdot K \lambda_p}{(E/\text{GeV})} \cdot X' = K/\gamma, \quad K = 0.934 \cdot (B/T) \cdot (\lambda_p/\text{cm}) \quad (2.11) \]

The maximum slope \( X'_0 \) and the maximum amplitude \( X_0 \) characterizes the trajectory of the electron beam. Both expressions are given by equation (2.11).

The characteristics of the synchrotron radiation from the wiggler are the same one as from the bending magnet, and so, the critical energy \( \varepsilon_c \) determines everything. To reach the same spectral range as from the bending magnets (given by \( \varepsilon_c \) (see equation 2.3)), the magnetic flux density within the wigglers must be the same as within the bending magnets, or for shifting the spectrum to higher photon energies, even higher. At CCRL, Daresbury Laboratory, UK, fields up to 2.5 Tesla could be reach with “Hybrid Design”, higher fields are possible with super conducting devices, at MAXLAB, Lund, Sweden fields up to 3 Tesla could be reached. In table (2.1) the data’s for the wigglers foreseen for SESAME are summarized.

<table>
<thead>
<tr>
<th>Type</th>
<th>( B_0 ) (T)</th>
<th>( \lambda_w ) (mm)</th>
<th>( N_w )</th>
<th>( L ) (m)</th>
<th>( K )</th>
<th>( X_0 ) (mm)</th>
<th>( X' ) (mrad)</th>
</tr>
</thead>
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<tr>
<td>W-100</td>
<td>2.0</td>
<td>100</td>
<td>24</td>
<td>2.4</td>
<td>18.7</td>
<td>0.061</td>
<td>3.8</td>
</tr>
<tr>
<td>W-120</td>
<td>2.5</td>
<td>120</td>
<td>20</td>
<td>2.4</td>
<td>28.0</td>
<td>0.109</td>
<td>5.7</td>
</tr>
<tr>
<td>W-60</td>
<td>3.5</td>
<td>60</td>
<td>30</td>
<td>1.8</td>
<td>19.6</td>
<td>0.038</td>
<td>4.0</td>
</tr>
</tbody>
</table>
The photon flux as well as the central intensity of the radiation emitted by the wiggler is the same as from the bending magnet but more intensive by a factor $N_p$, where $N_p$ is the number of poles within the wiggler. The number of $N_p$ is according to table (2.1) roughly 50 (2 times $N_w$). The photon flux as well as the flux density emitted from these wiggler at SESAME is presented in the figure (2.8) and (2.9). The comparison to the flux of the bending magnet shows, that the wiggler is roughly of a factor 50 more intensive, for the flux density it is a factor 20 to 25.

The calculation of the brilliance of wigglers needs to take into account the depth-of-fields, i.e. the contribution to the apparent source size from different poles. The expression for the brilliance of wigglers is:
$Br_{wi} = \langle \frac{d^2\Phi}{d\Phi d\psi} (\psi = 0) \rangle \cdot \sum_{n=-N/2}^{N/2} \frac{1}{2\pi} \cdot \exp \left\{ \frac{-1}{2} \left[ \frac{X_0^2}{\sigma_x^2 + z_n^2 \sigma_x^2} \right] \right\} \div \left[ \left( \frac{\sigma_y^2 + z_n^2 \sigma_x^2}{\sigma_y^2} \right) \left( \frac{\varepsilon_y^2}{\sigma_y^2} + \sigma_y^2 + z_n^2 \sigma_y^2 \right) \right]^{1/2}$ \hspace{1cm} (2.12)

where:

$z_n = \lambda_p \left( n + \frac{1}{4} \right)$ \hspace{1cm} (2.13)

$\sigma_x$, $\sigma_x'$, $\sigma_y$ and $\sigma_y'$ are the rms transverse size and angular divergence of the electron beam at the centre of an insertion straight section ($\sigma_x = \sigma_y = 0$). This means that the brilliance of the wiggler, calculated according to equation (2.12), is normalized to the middle of the straight section.

The exponential factor in equation (2.12) arises because the wigglerse have two points, separated by $2^*X_0$ according to equation (2.11). The sum in equation (2.12) goes over all poles of the wiggler. As already discussed under the radiation of the bending magnets the factor $(\varepsilon_y/\sigma_y)$ is at least a factor 10 smaller than the cross section $\sigma_y$, and can be neglected. The expression $z_n^* \sigma_y'$ is the increase of the source size from the centre of the insertion device.

Instead of normalizing the brilliance to the centre of the straight section, the cross sections of the beam size $\sigma_x(n)$ and $\sigma_y(n)$ at the position of the different poles can be used, with the result that the expression for the brilliance will be simpler:

$Br_{Wig} = \langle \frac{d^2\Phi}{d\theta d\psi} (\psi = 0) \rangle \cdot 2 \cdot \sum_{n=-N/2}^{N/2} \frac{1}{2\pi} \cdot \exp \left\{ \frac{-1}{2} \left[ \frac{X_0^2}{\sigma_x^2(n)} \right] \right\} \div \sigma_x(n) \cdot \sigma_y(n)$ \hspace{1cm} (2.14)

The brilliance of the three types of wiggleres given in table (2.1) are presented in figure (2.10). The brilliance is determined by the flux and the cross sections of the beam, because of the cross sections at the locations of the wiggleres are 3 or 4 times larger as in the bending magnet, the brilliance of is of a factor 20 more intensive as of the bending magnet.

The brilliance of a wiggler is according to equation (2.14) inversely proportional to the cross section of the beam and not to the emittance. The beam cross section can by manipulated with the beta function in the storage ring. The so called “mini beta sections” are leading to a small beam size. The dependency of the wiggler brilliance upon the beta function at the location of the wiggler is given in figure (2.11), which is the inverse of the dominator of equation (2.14) for different beta functions in the middle of the straight section. For high beta functions (4 to 22 m/rad) the cross section of the beam doesn’t change very much and the brilliance is according to figure (2.11) proportional to the length of the wiggler. For small beta functions (0.4 and 0.6 m/rad) the brilliance of the wiggler will saturate, because the cross section of the beam in the outer parts of the wiggler gets high and the contribution to the brilliance is small and can therefore be neglected.
In order to optimise the brilliance of the wiggler radiation the beta functions in the middle of the straight sections should be as small as possible (mini-beta-section) and the length of the wiggler has only to be around 2 to 3 m, because the outer regions of the wiggler don’t have any significant contribution to the brilliance.
2.4 Radiation from an Undulator

The opening angle $\sigma_\gamma$ of the synchrotron radiation from the bending magnet at the critical photon energy $\varepsilon_c$ is according to equation (2.5) roughly $0.655/\gamma$ or $1/\gamma$. The maximum slope of the electron trajectory in a wiggler is $X' = K/\gamma$. For values of $K$ in the range between 1 to 2 the deflection angle in a wiggler is within the opening angle of the synchrotron radiation. For this special case the radiation from different periods interferences coherently, thus producing sharp peaks with the result of completely different characteristics. This radiation, as symbolized in figure (2.12), is called undulator radiation and the corresponding insertion devices are undulators.

![Figure 2.12: The characterization of the undulator radiation. L is the period length of the undulator](image)

Undulators are insertion devices like the wigglers but with a smaller $K$-values (between 1 and 3). A principle set up of an undulator is given in figure (2.13). The blocks with the different colours symbolize the permanent magnets with the direction of the magnetic field. The period length is indicated with $\lambda$.

![Figure 2.13: The arrangement of magnets within an undulator. “Red”, “Green”, “Blue” and “Yellow” are permanent magnets.](image)
The undulator emits radiation only at characteristic photon energies:

\[ \varepsilon_n = 0.949 \text{KeV} \cdot (E / \text{GeV})^2 \cdot \frac{n}{(\lambda_{Und} / \text{cm}) \cdot \left(1 + K^2 / 2\right)} \]  

(2.15)

with the bandwidth:

\[ \Delta \varepsilon_n = \frac{1}{\varepsilon_n} \cdot \frac{1}{n N_{Und}} \]  

(2.16)

where:

- \( n \) = Harmonic number (\( n = 1, 3, 5, 7, \ldots \)).
- \( N_{Und} \) = Number of periods.
- \( \lambda_{Und} \) = Period length of the undulator.
- \( K \) = Deflection parameter (see Equation (2.10)).

The opening angle of the undulator radiation cone as indicated in figure (2.12) is:

\[ \sigma_\gamma = \frac{1}{\gamma} \cdot \sqrt{\frac{1 + K^2 / 2}{2 N n}} \]  

(2.17)

For the illustration of the characteristics of the undulator radiation, the following example shall be used:

- \( E = 2 \text{ GeV}, \lambda_{Und} = 40 \text{ mm}, K = 2 \text{ and } N = 50 \).
- \( \varepsilon_1 = 0.316 \text{ keV} \) and \( \varepsilon_9 = 2.85 \text{ keV} \).
- \( \Delta \varepsilon_1 = 6.32 \text{ eV} \) and \( \Delta \varepsilon_9 = 6.32 \text{ eV} \).
- \( \sigma_\gamma(1) = 0.0443 \text{ mrad} \) and \( \sigma_\gamma(9) = 0.0148 \text{ mrad} \).

The flux of the undulator radiation within the cone of the harmonics is given by (in practical units):

\[ \Phi_{Und}(n, K) = 1.432 \cdot 10^{14} \cdot N_{Und} \cdot (I / A) \cdot Q_n(K) \text{ [Photons/(s 0.1\%BW)]} \]  

(2.18)

According to equation (2.18) the flux of the undulator radiation is proportional to the number of periods, the current and the function \( Q_n(K) \). It is independent of the energy of the electrons. To reach a high photon flux (also for higher harmonics), the deflection parameter \( K \) should be in the range of 2 to 3. The photon spectrum is according to equation (2.15) proportional to the square of the energy and inversely proportional to the period length \( \lambda \). In order to reach x-ray radiation the energy has to be in the range of 4 to 6 GeV with a period length of 40 mm. The gap would be 11 mm and the magnets of the undulator are outside the vacuum (figure (2.13)). Smaller period lengths can be reached only by reducing the gap; this is possible by putting the undulator inside the vacuum. This type of undulator is called “In Vacuum Undulator” (figure (2.15)). The smallest period lengths can be reached with super conducting magnets. These types are called “Mini-Undulator” (figure (2.16)).

The photon flux of these undulators are presented in the figures (2.14) – (2.16). The comparison of these figures show that the highest flux and the largest spectrum can be reached with small gaps.
Figure 2.14: The photon flux emitted from an undulator at SESAME. The parameters are: $\lambda = 40\,\text{mm}$, gap = $11\,\text{mm}$, $k = 1.4 - 2.8$, $L = 2.4\,\text{m}$, $E = 2.5\,\text{GeV}$, $\varepsilon = 23.7\,\text{nm}$, coupl. = $2\,\%$.

Figure 2.15: The photon flux emitted from an undulator at SESAME. The parameters are: $\lambda = 25\,\text{mm}$, gap = $7\,\text{mm}$, $k = 1.0 - 2.0$, $L = 1.4\,\text{m}$, $E = 2.5\,\text{GeV}$, $\varepsilon = 23.7\,\text{nm}$, coupl. = $2\,\%$.

Figure 2.16: The photon flux emitted from an undulator at SESAME. The parameters are: $\lambda = 14\,\text{mm}$, gap = $5\,\text{mm}$, $k = 1.0 - 2.0$, $L = 1.4\,\text{m}$, $E = 2.5\,\text{GeV}$, $\varepsilon = 23.7\,\text{nm}$, coupl. = $2\,\%$.

The peak intensity on axis of the nth harmonic of the undulator radiation is given by (in practical units [photons / (s 0.1% BW m²)]):

\[ I_n = \frac{4\pi\varepsilon_0 c^2}{\lambda^3} \frac{1}{\sin^2 \theta} \frac{1}{n^2} \frac{1}{(1 - k^2)^2} \]

where $\varepsilon_0$ is the permittivity of free space, $c$ is the speed of light, $\theta$ is the angle of incidence, $n$ is the harmonic number, and $k$ is the wave number.
\[ \frac{d^2 \Phi_{\text{Und}}}{d \theta d \psi}(\theta = \psi = 0) = 1.744 \cdot 10^{14} \cdot N_{\text{Und}}^2 \cdot \left( \frac{E}{\text{GeV}} \right)^2 \cdot \left( \frac{I}{A} \right) \cdot F_n(K) \] (2.19)

The peak intensity is proportional to the electron current, the radiation function \( F_n(K) \) and the square of the period number. The opening angle of the radiation cone is inversely proportional to the energy, and therefore the peak intensity according to Equation (2.19) is proportional to the square of the energy.

The brilliance of the undulator radiation is given by:

\[ B_{\text{Und}} = \frac{\Phi_{\text{Und}}(\theta = \psi = 0)}{4\pi \int \sigma \cdot \sigma' \cdot d\theta d\psi} \] (2.20)

where

\[ \sigma = \sqrt{\sigma_x^2 + \sigma_y^2}, \quad \sigma' = \sqrt{\sigma_{x'}^2 + \sigma_{y'}^2} \] (2.21)

\[ \sigma_x = \sqrt{\beta_{x,y}} \cdot \sigma_{x,y}, \quad \sigma'_{x,y} = \sqrt{\beta_{x,y}} \cdot \sigma_x, \quad \sigma_r = \frac{1}{4\pi} \sqrt{\lambda L}, \quad \sigma'_{r} = \sqrt{\frac{\lambda}{L}} \] (2.22)

Inserting \( \lambda \) according to Equation (2.15), \( \sigma_r \) and \( \sigma'_{r} \) are given by:

\[ \sigma_r = \frac{\lambda_{\text{Und}}}{4\pi\gamma} \sqrt{\frac{1 + K^2 / 2}{2n}} \cdot N, \quad \sigma'_{r} = \frac{1}{\gamma} \sqrt{\frac{1 + K^2 / 2}{2nN}} \] (2.23)

For the above mentioned undulator the cross sections \( \sigma_r \) and divergences \( \sigma'_{r} \) according to equation (2.23) are summarized in the table (2.2).

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_r [\mu m] )</td>
<td>7.05</td>
<td>4.07</td>
<td>3.15</td>
<td>2.66</td>
<td>2.35</td>
</tr>
<tr>
<td>( \sigma'_{r} [\mu rad] )</td>
<td>44.3</td>
<td>25.6</td>
<td>19.8</td>
<td>16.7</td>
<td>14.8</td>
</tr>
</tbody>
</table>

For the electron, the cross sections and divergences are: \( \sigma_x = 365 \mu m, \sigma_y = 28.7 \mu m, \sigma_{x'} = 46 \mu rad, \sigma_{y'} = 11.6 \mu rad \). So, \( \sigma_r \) can be neglected for the calculation of the brilliance.

The brilliance of the three types of undulators: U40, U25 (In Vacuum) and U14 (Mini-Undulator) are presented in figure (2.17) – (2.19). The comparison of these figures show that the highest brilliance can be reached with small gaps. At SESAME it is only possible to reach the 12 keV photons with “In Vacuum” undulators and the 20 keV with “Mini-Undulators”.

In figure (2.20) the photon fluxes and in figure (2.21) the photon brilliances emitted within the bending magnet, the wigglers and the undulators have been compiled.
Figure 2.17: The brilliance emitted from an undulator at SESAME. The parameters are: $\lambda = 40$ mm, gap = 11 mm, $k = 1.4 - 2.8$, $L = 2.4$ m, $E = 2.5$ GeV, $\varepsilon = 23.7$ nm, coupl. = 2 %.

Figure 2.18: The brilliance emitted from an undulator at SESAME. The parameters are: $\lambda = 25$ mm, gap = 7 mm, $k = 1.0 - 2.0$, $L = 1.4$ m, $E = 2.5$ GeV, $\varepsilon = 23.7$ nm, coupl. = 2 %.

Figure 2.19: The photon flux emitted from an undulator at SESAME. The parameters are: $\lambda = 14$ mm, gap = 5 mm, $k = 1.0 - 2.0$, $L = 1.4$ m, $E = 2.5$ GeV, $\varepsilon = 23.7$ nm, coupl. = 2 %.
Figure 2.20: The photon flux emitted from the bending magnet, wigglers and undulator at SESAME. The details of the insertion devices are given within the text.
Figure 2.21: The photon brilliance emitted from the bending magnet, wigglers and undulator at SESAME. The details of the insertion devices are given within the text.

Reference