INFLUENCE OF THE VACUUM CHAMBER LIMITATION ON DYNAMIC APERTURE CALCULATIONS

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Abstract
In a storage ring the evaluation of the dynamic aperture taking into account the vacuum chamber limitation is more accurate and may display nonlinearities that could not be seen in the conventional absolute dynamic aperture calculations. This has been demonstrated in SESAME case where taking into account the vacuum chamber uncovered the seriousness of a 5th order resonance mainly when high order multipoles were introduced to the lattice. The destructive effect of the 5th order resonance has been avoided by changing the fractional part of the tunes. The results are crosschecked using two tracking codes and verified using the Frequency Map Analysis technique.

INTRODUCTION
The dynamic aperture is the transverse area in the x-z plane (x for horizontal and z for vertical plane) in which the particle betatron motion is stable. It is defined by the maximum initial phase space amplitudes (x(0), p_x(0), z(0), p_z(0)) with which the tracked particle doesn’t get lost for enough number of turns with respect to the interesting time scale as the damping time for the electrons [1]. It is a local lattice parameter where its horizontal and vertical dimensions at some longitudinal position s depend on the optical functions there. In the linear approximation it is transformed using the relation

\[ A_{x,z}(s) = \sqrt{\beta_{x,z}(s) / A_{0x,z}}, \]

with \( \beta_{x,z}(s) \) the s-dependent beta function, \( A_{0x,z} \) and \( A_{0x,z} \) are beta function and dimensions of the dynamic aperture at the calculation point \( s = 0 \).

In BETA [2] and TRACY-II [3] tracking codes, used in this study, the particle is tracked in different ways. In BETA code the particle coordinates are defined by a column matrix with the components \((x, x', z, z', l, \delta)\) where \(x, z\) the horizontal and vertical transverse positions, \(x' = dx /ds\) and \(z' = dz /ds\) are the angles, \(l\) and \(\delta\) are the variations in path length and the relative momentum deviation of the test particle from the synchronous one, whereas the \(7^{th}\) component \(l\) is used to represent the effect of a kick on the trajectory. When dealing with the 2nd order formalism, the column vector is extended by adding the 2nd order components [4]. The particle tracking is done using the 1st order and 2nd order transfer matrices [4]. In TRACY-II code the particle motion is described by the canonical coordinates \((x, p_x, z, p_z, l, \delta)\) with \(x, z\) the horizontal and vertical transverse positions and \(p_x, p_z\) are their horizontal and vertical conjugate momenta. The tracking is done using the 2nd order or 4th order symplectic integrators where the particle motion is symplectic [5].

Conventionally, the nonlinear beam dynamics is represented by the absolute dynamic aperture calculations where a wide excursion space is offered for the particle [6]. The oscillating particle is considered unstable when it exceeds that space. For more realistic estimation for the dynamic aperture, the vacuum chamber should be included in the calculations since it defines the realistic physical limits to the particle excursion amplitude. In this case, a particle passing close to the chamber borders with nonlinear motion may get lost at the chamber limitation and considered as unstable particle, while it can be described as a stable one if it had larger space to oscillate in as in the absolute dynamic aperture case. In this sense the vacuum chamber may participate in defining the “chamber-limited” dynamic aperture [7]. The importance of including the vacuum chamber can be seen more clearly if the particle nonlinear motion is excited by the effect of high order multipoles [8] for example.

THE DYNAMIC APERTURE WITH VACUUM CHAMBER
In the nonlinear optimization of SESAME storage ring lattice, the vacuum chamber with dimensions \(x = \pm 35 \text{ mm}\) for horizontal half-aperture and \(z = \pm 15 \text{ mm}\) for the vertical one was included in the dynamic aperture calculations. The vacuum chamber was introduced in TRACY-II as a transverse physical limitation at the entrance and exit of each element in the lattice. The elements of the lattice are divided into many slides, for each the vacuum chamber limitations are introduced. The bending magnets and quadrupoles are the most interesting ones in this consideration. In BETA code, the vacuum chamber was represented by horizontal and vertical scrapers placed at the highest values for beta functions \(\beta_t\) and \(\beta_c\).

The presented nonlinear calculations are done on an Optics with working point \((Q_x = 7.23, Q_z = 5.19)\) [9] and are evaluated by tracking the particle for 1000 turns starting from the middle of the Long straight section where \(\beta_t = 12.31 \text{ m}\) and \(\beta_c = 3.13 \text{ m}\). The maximum values for \(\beta_t\) and \(\beta_c\) in this optics are \(\beta_{t\text{max}} = 12.807 \text{ m}\) in the middle of the focusing quadrupoles at the ends of the long straight sections and \(\beta_{c\text{max}} = 21.35 \text{ m}\) in the middle of the bending magnets as shown by Fig. 1. The vacuum chamber vertical half-aperture, \(z = 15 \text{ mm}\), in the bending
magnet yields a vertical physical aperture $\Delta z = 5.74$ mm (at $x = 0$) at the calculation point. The presented calculations are done for chromaticities corrected to zero in both planes.

When the dipole high order multipoles, listed in Table 1, were included into the calculations the size of the absolute and chamber-limited dynamic apertures became as shown in Fig. 3.

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### Table 1: Dipole high order multipoles.

<table>
<thead>
<tr>
<th>High Order Component</th>
<th>$(\Delta B_z / B)$</th>
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<tbody>
<tr>
<td>Sextupole</td>
<td>$2.42 \times 10^{-4}$</td>
</tr>
<tr>
<td>Octupole</td>
<td>$4.70 \times 10^{-5}$</td>
</tr>
<tr>
<td>Decapole</td>
<td>$-3.09 \times 10^{-5}$</td>
</tr>
<tr>
<td>Dodecapole</td>
<td>$-1.36 \times 10^{-5}$</td>
</tr>
<tr>
<td>14-pole</td>
<td>$-1.17 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Figure 3(left) indicates that the absolute dynamic aperture is still enough larger than the physical one, consequently the high order multipoles seem tolerable by the optics. But when the vacuum chamber is included in the calculations in Fig. 3(right), we can see that these high order multipoles amplify the two cuts to a level that cannot be accepted resulting in a dynamic aperture much smaller than the physical one. Hence these high order multipoles are not tolerable by the optics, contradicting the indication given by Fig. 3(left). It can be noticed that the two cuts also have been shifted outward from the center. This is due to the distorted tune shifts with $x$ and $z$-amplitudes. Propagation of the two vertical cuts down through the dynamic aperture indicates that the driving resonance is excited causing higher amplitudes for the particle nonlinear vertical oscillations so that the particle gets lost on the upper chamber limitation at lower vertical heights.

This explanation is more clarified by Fig. 4 which shows behavior of the vertical oscillation amplitude of the particle versus $x$-position at $z = 4.8$ mm without and with high order multipoles. The blue lines represent the vertical physical aperture $\Delta z = \pm 5.74$ mm at the calculation point.

**Applying High Order Multipoles**
The gradually increasing vertical amplitude with \( x \) in the left sides of Fig. 4 explains the dynamic aperture degradation in the left hand side of Figs. 2(right) and 3(right). The two drastic increments in vertical amplitude at \( x \approx \pm 21 \) mm which are amplified by high order multipole effect stand behind the two seen cuts since they cause the particle to exceed the vertical acceptance of the vacuum chamber.

The above investigation shows that introducing the vacuum chamber limitations into the calculations was a simple tool to uncover the inner nonlinearity in SESAME dynamic aperture which couldn’t be seen in case of absolute dynamic aperture calculations.

**CONCLUSION**

This study showed that including the vacuum chamber limitation in the dynamic aperture calculations could be a simple tool, other than the complicated FMA technique, to uncover the nonlinearity in the inner structure of the dynamic aperture. In SESAME case it revealed a seriousness of existing 5\(^{th}\) order resonance, mainly when high order multipoles are included, something which couldn’t be noticed in case of absolute dynamic aperture calculations.

**REFERENCES**